M — Marvelous Marathon

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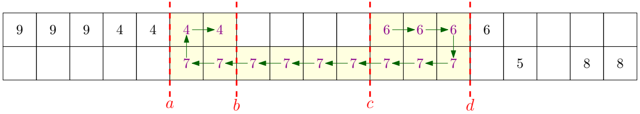
Formalized version of problem

Find integers $0 \leq a \leq b \leq c \leq d \leq m$ such that $2(b-a)+(c-b)+2(d-c)=x$.



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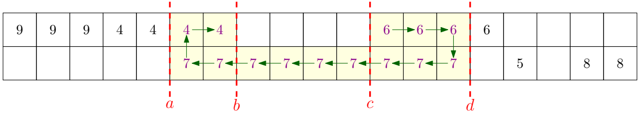
$$\text{Maximize} \left\{ \begin{array}{c} \text{sum of cells of one row in range } [a,d) \\ \text{plus} \\ \text{sum of cells of other row in ranges } [a,b) \text{ and } [c,d) \end{array} \right\}$$



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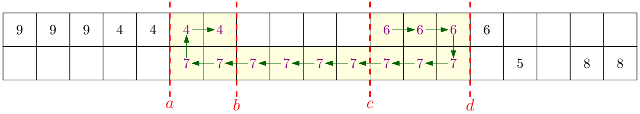
Solution outline

1. m is very large, cannot loop over all cells



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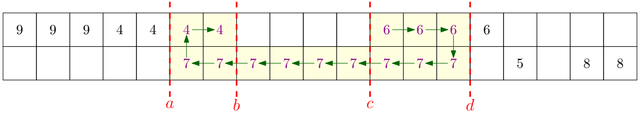
Solution outline

1. m is very large, cannot loop over all cells
2. Large parts of grid look the same because only n segments



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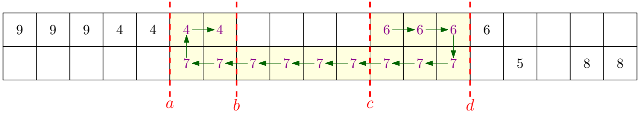
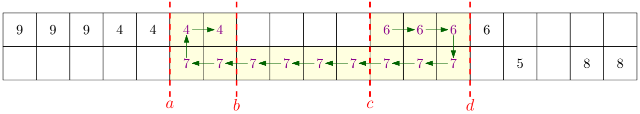
1. m is very large, cannot loop over all cells
2. Large parts of grid look the same because only n segments
3. Separately handle three main cases: 0, 1 or 2 U-turns



Solution outline

1. m is very large, cannot loop over all cells
2. Large parts of grid look the same because only n segments
3. Separately handle three main cases: 0, 1 or 2 U-turns
4. We focus here only on the hardest case with 2 U-turns.



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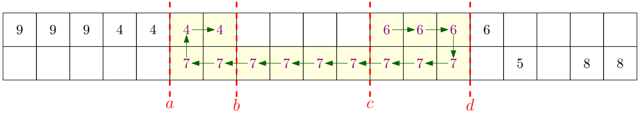
Insight 1

1. We can assume solution has the gap in the lower half
   - Run solution again on flipped input to cover opposite case



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1. We can assume solution has the gap in the lower half
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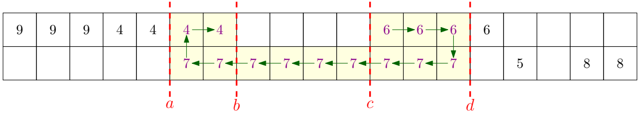
2. There is an optimal solution where a or d is at a segment endpoint (or 0 or m). Otherwise we could decrease (or increase) both a and d with 1 until either



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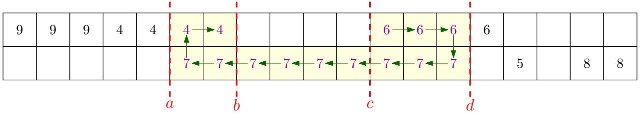

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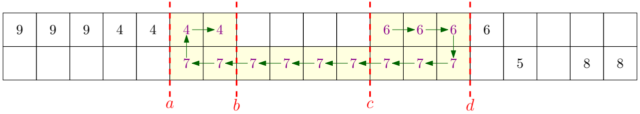

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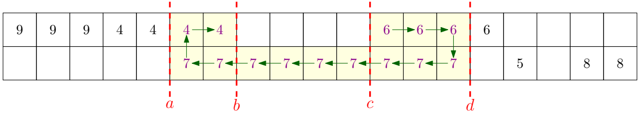
   - $d = c$ or $a = b$, in which case we end up with the 1 U-turn case (handled separately, left as an exercise!)
3. We can assume a is the endpoint
   - Run solution again on reversed input to cover opposite case.



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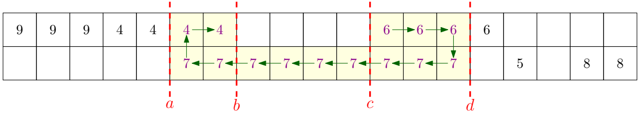
Insight 2

1. There is an optimal solution where b or c is at a segment endpoint, for the same reasoning as before (except that this time we would show it by shifting b or c).



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2. We end up with two cases to consider:
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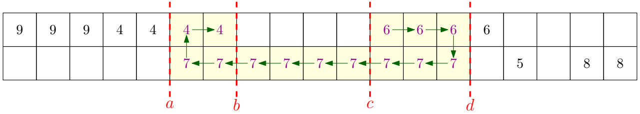

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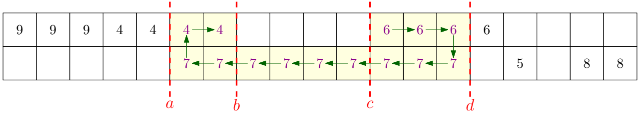
   - a and b are segment endpoints
   - a and c are segment endpoints
3. Will focus on the first of these; the other must also be solved, but is done in a very similar fashion.



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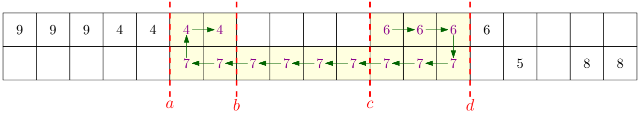
Sliding

1. Fix some a and b. ($O(n^2)$ possible choices.)



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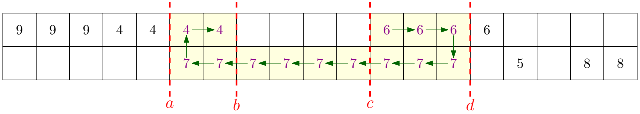
2. Set $c = b$ (or $c = b + 1$ if x is odd) and $d = a + \lceil x/2 \rceil$



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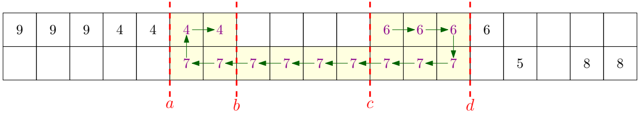

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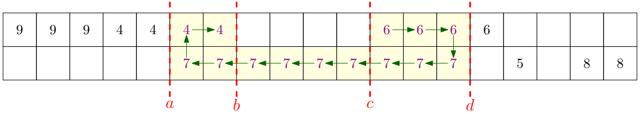

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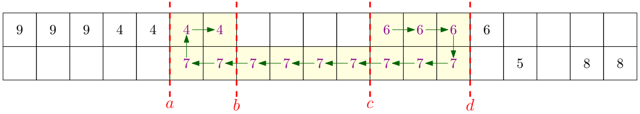

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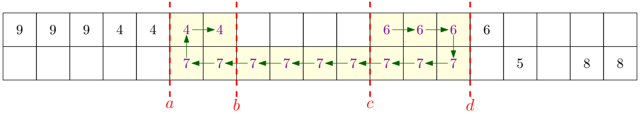
    - Repeat until c reached the end of the road.



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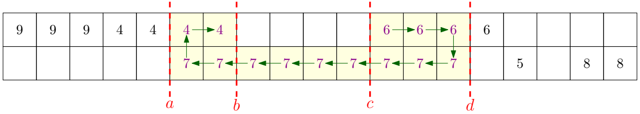

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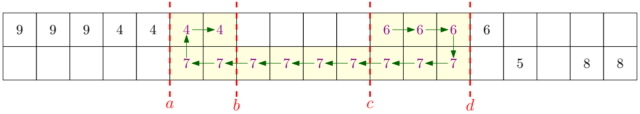

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Statistics at 4-hour mark: 0 submissions, 0 accepted