Problem I
Intrinsic Interval

Submits: 42
Accepted: at least 1

First solved by: Jagiellonian 1
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(Hlembotskyi, Stokowacki, Zieliński)
02:10:47

Author: Gustav Matula

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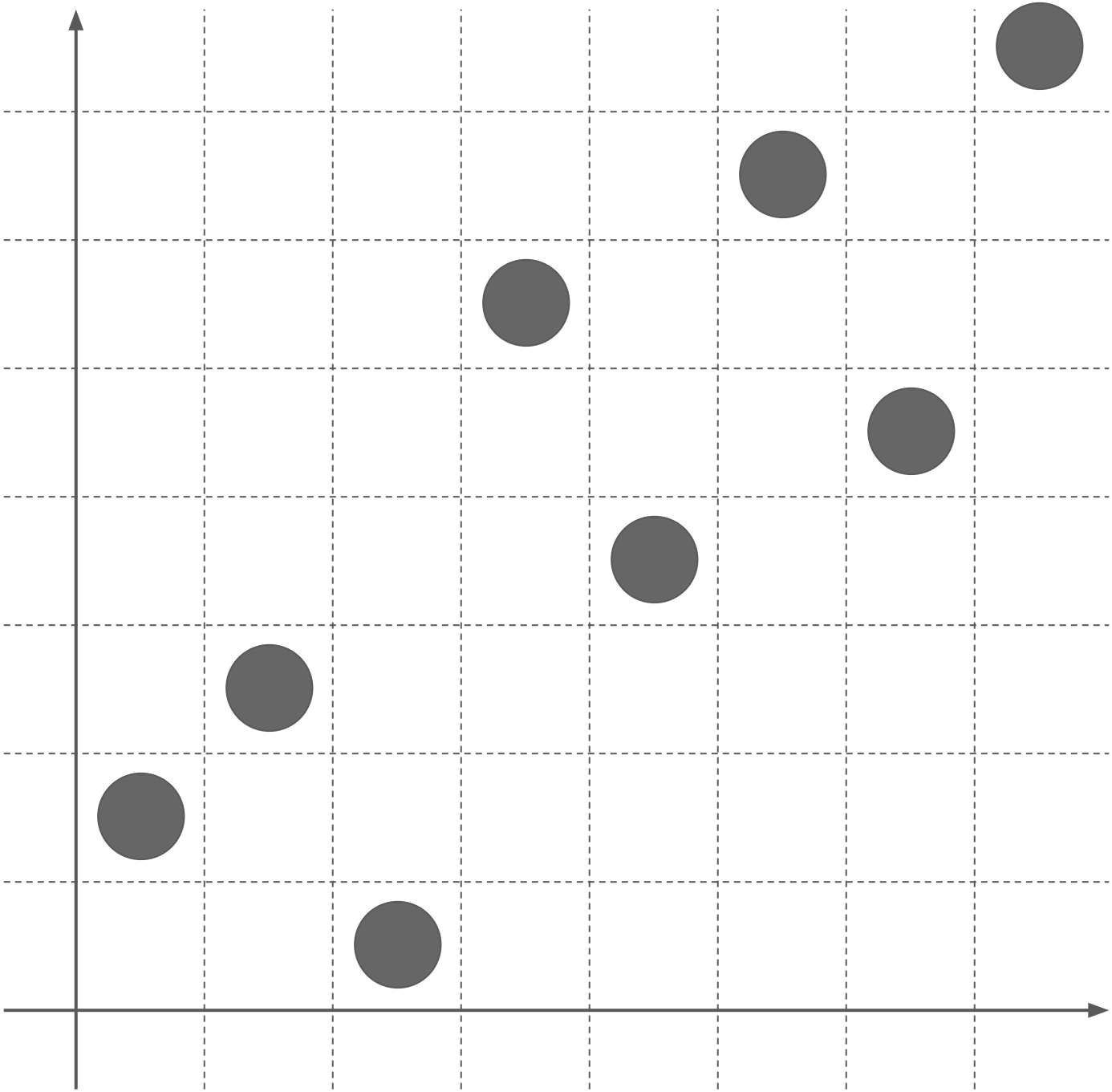
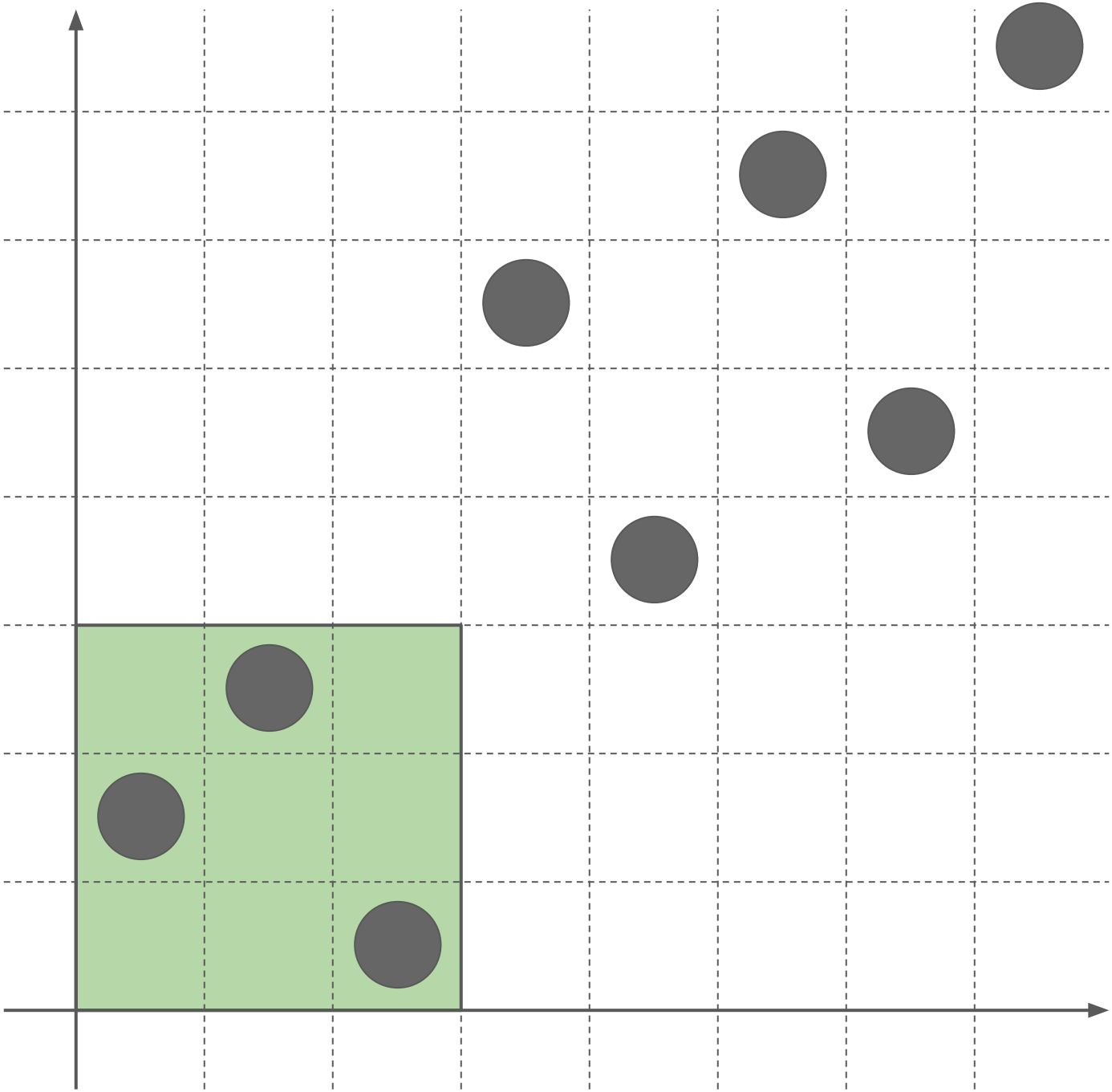
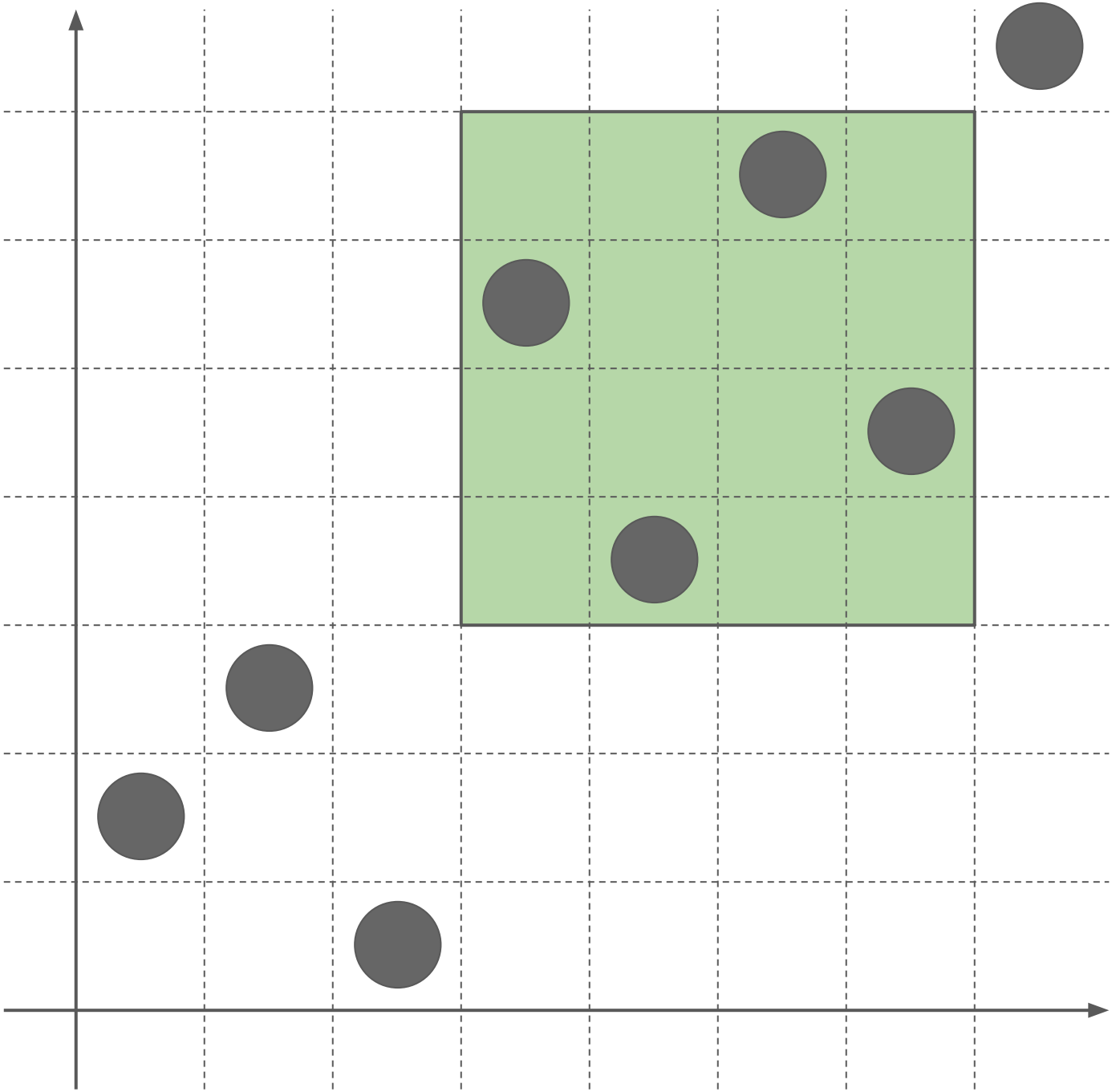
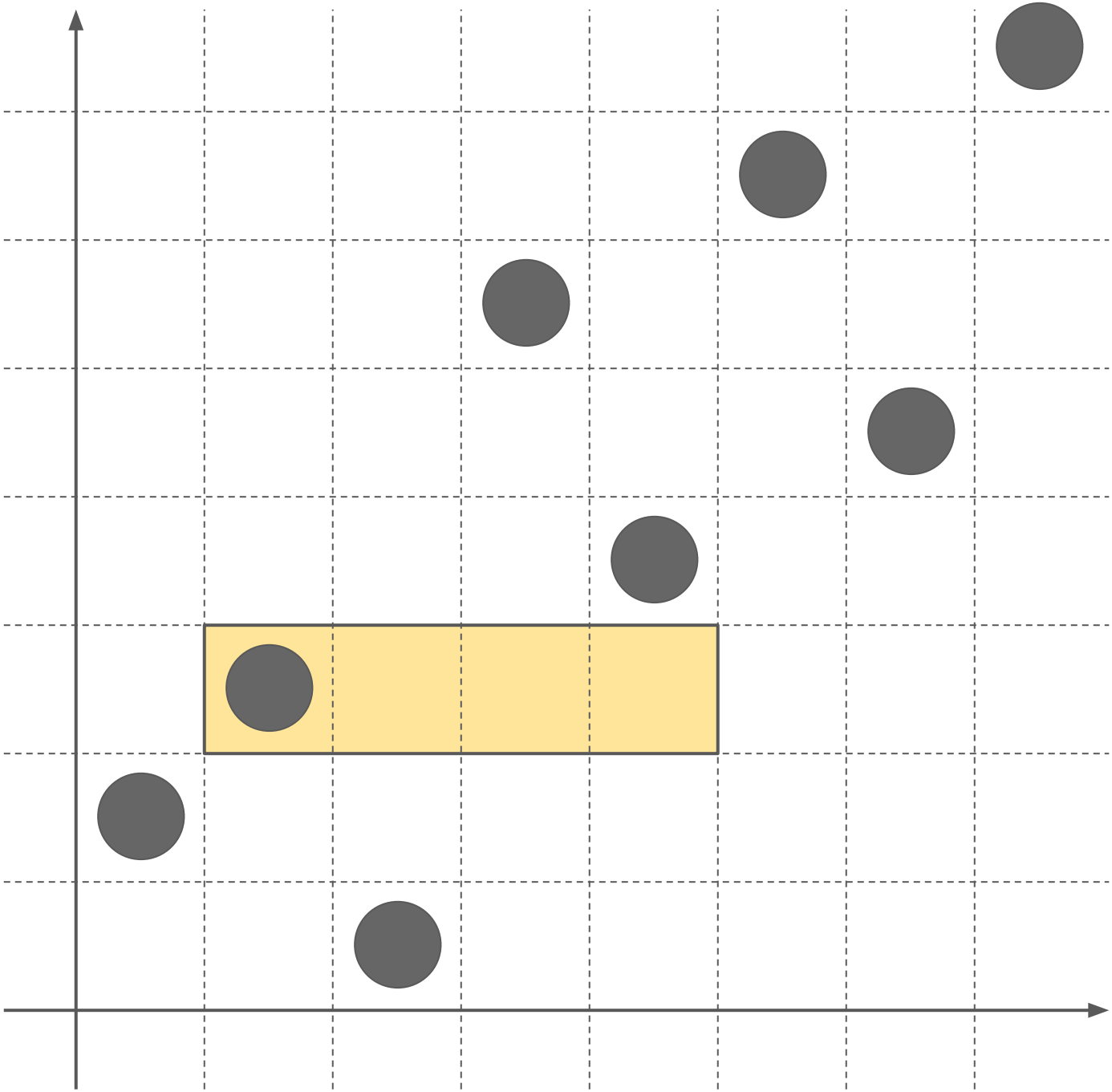
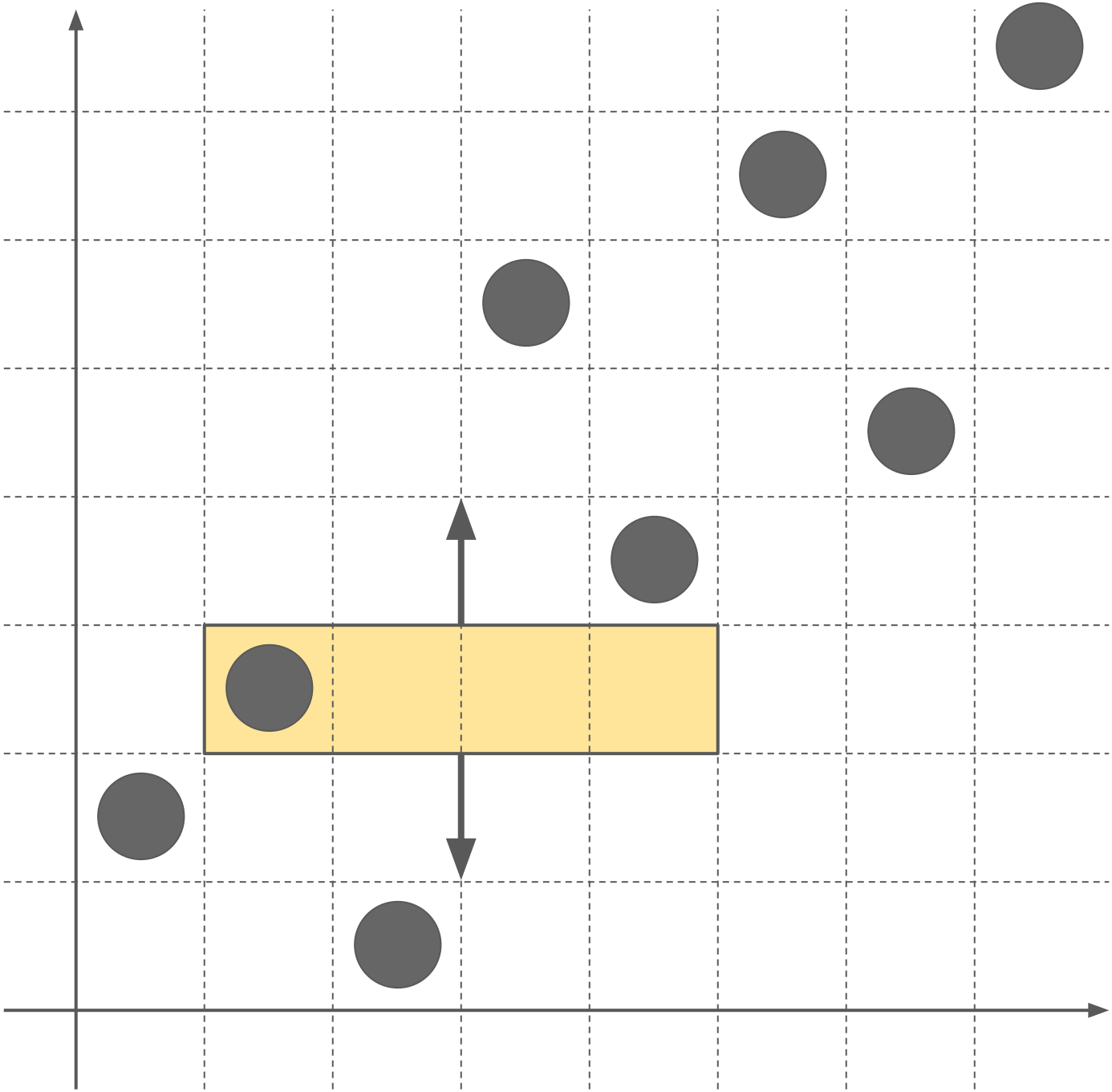
To see how we could expand the subsequence into the shortest enclosing interval, let's visualize the permutation in two dimensions.

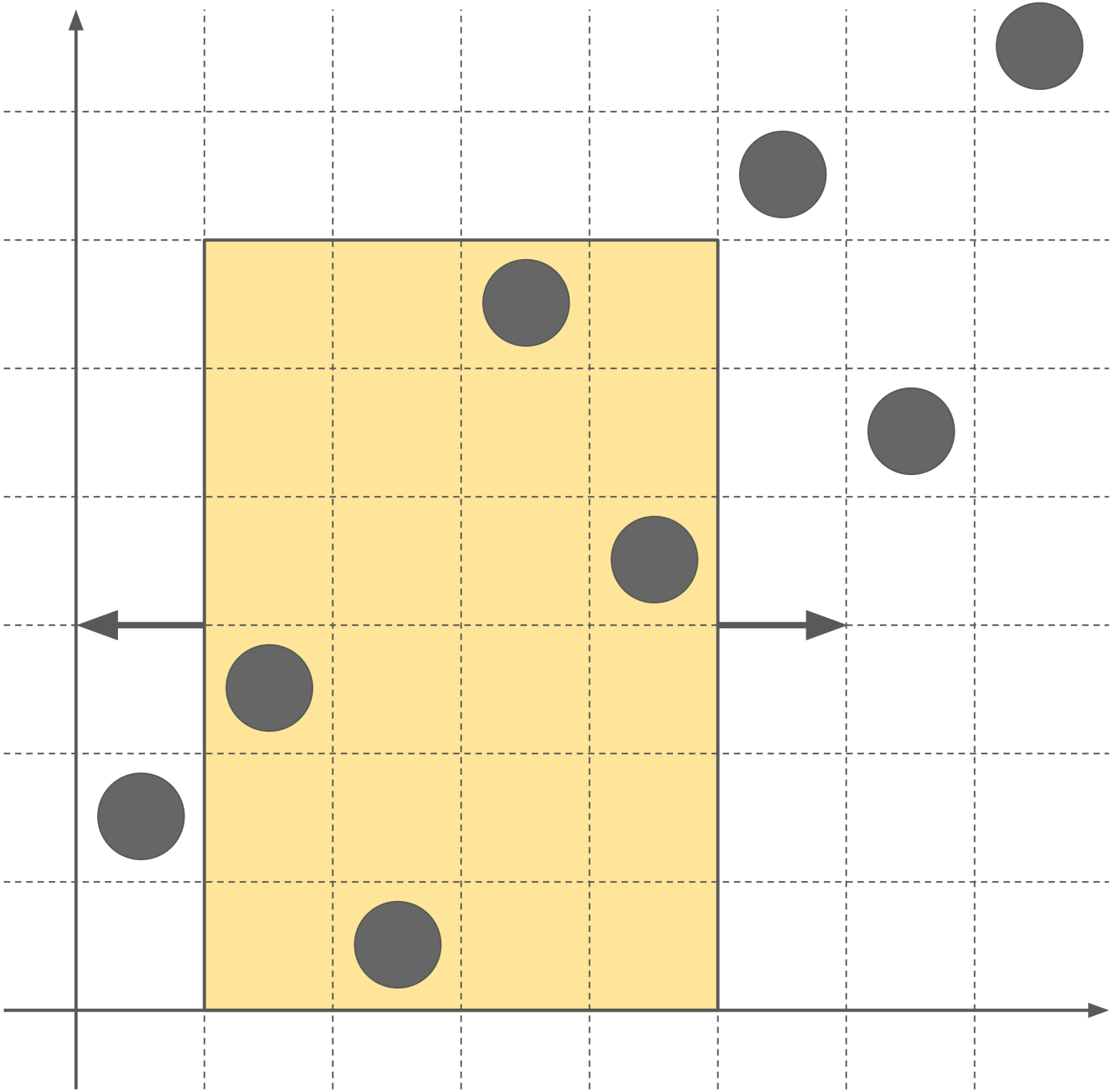


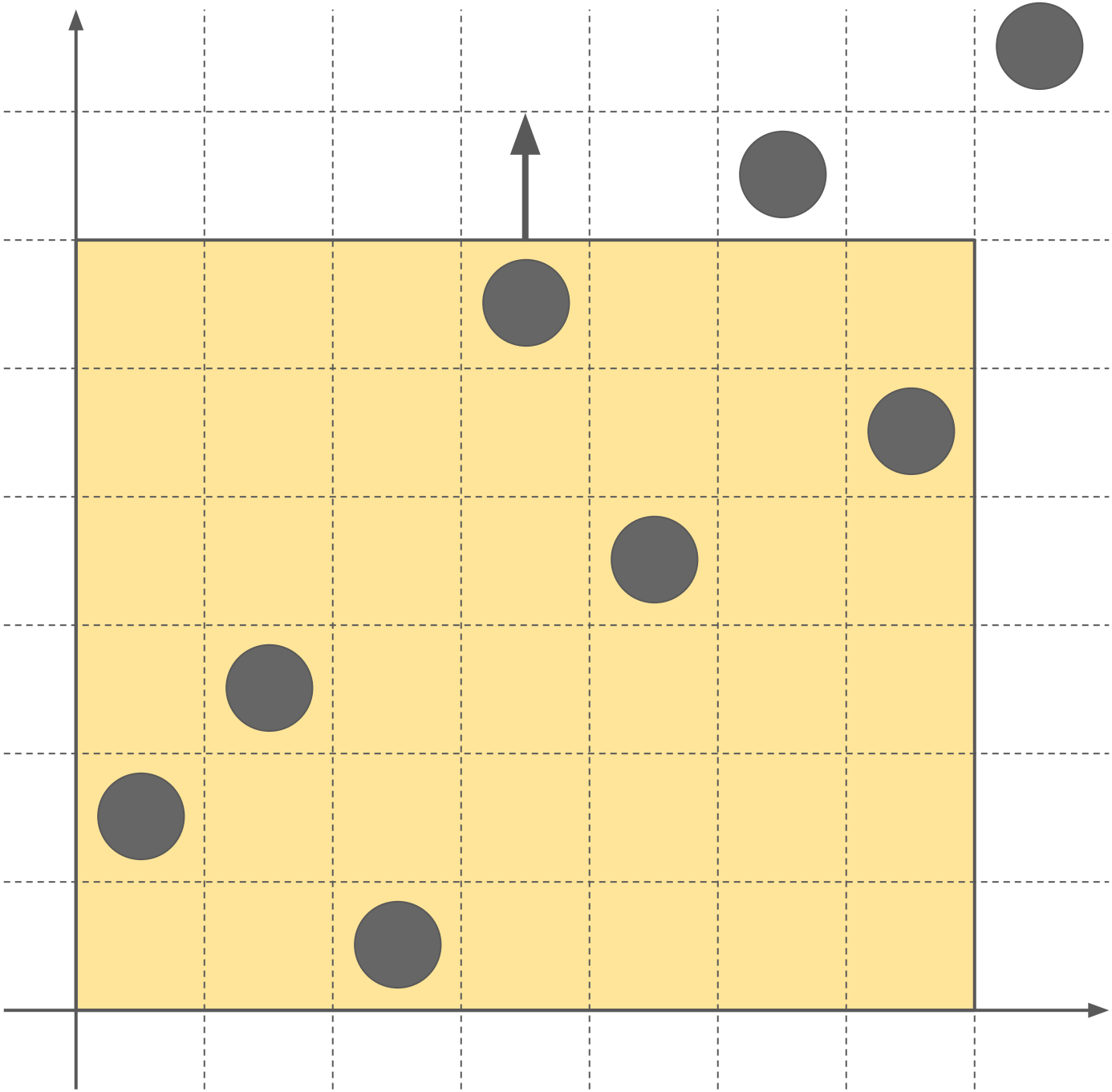


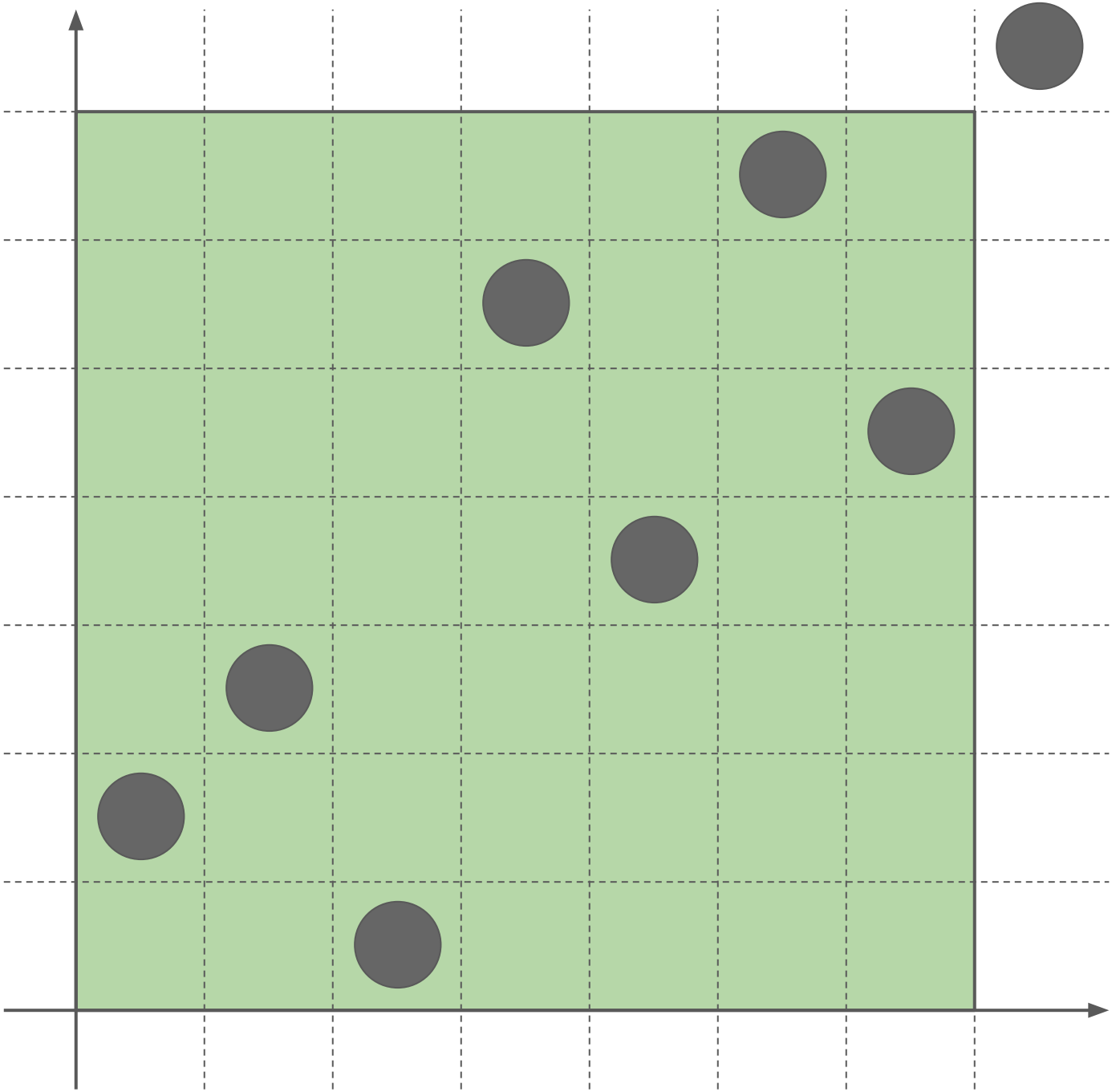












With careful implementation of the algorithm, it is possible to expand a subsequence [a, b] to an enclosing interval [x, y] in O(|y - x| - |b - a|).

However, that's too slow for this problem.

Instead, we'll develop divide and conquer algorithm to answer all queries at once.

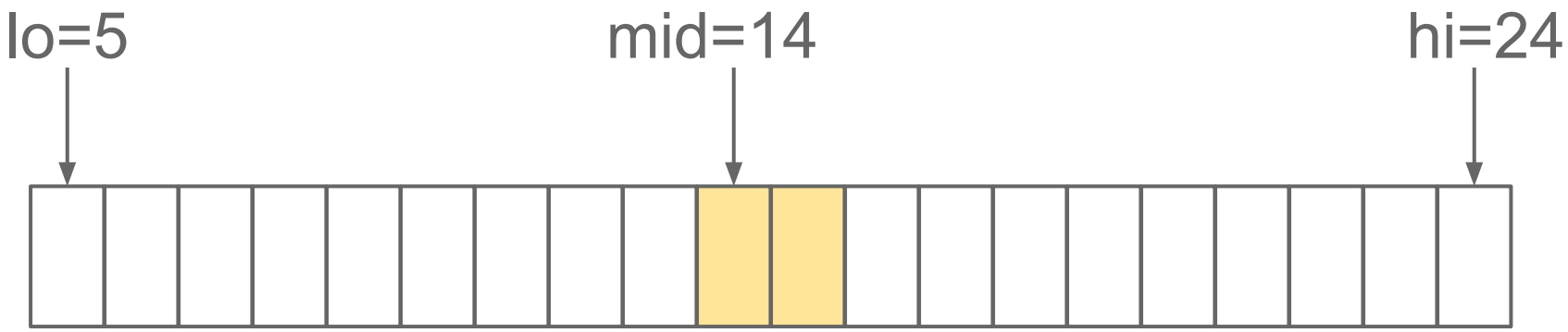
We initialize the result for each query with interval [1, n] and then we'll try to improve it.

Improve(queries, lo, hi) will try to improve each query in queries by considering intervals completely within [lo, hi] window.

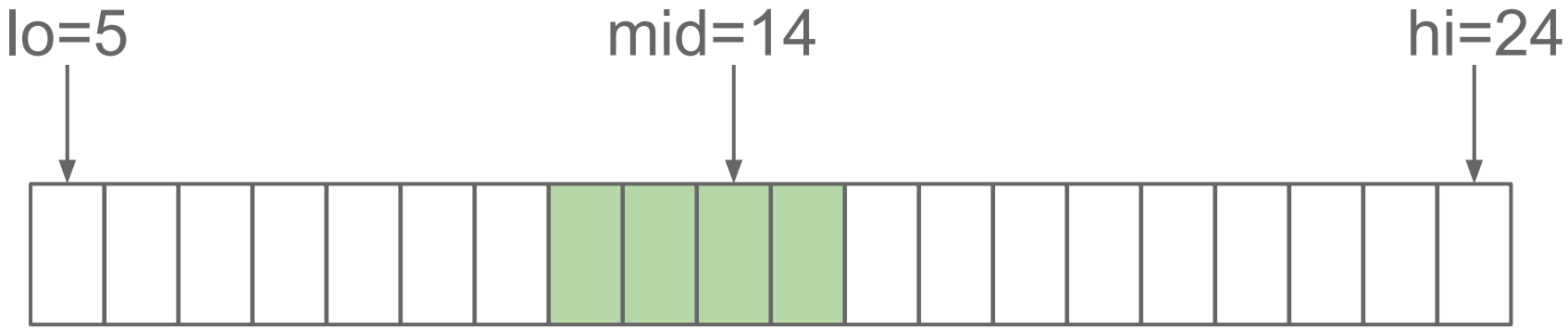
```
Improve(queries, lo, hi):
  if lo == hi: return
  mid = (lo + hi) / 2
  Improve([q in queries where q.b <= mid], lo, mid)
  Improve([q in queries where q.a > mid], mid + 1, hi)

  ImproveViaMid(queries, lo, mid, hi)
```

ImproveViaMid considers all intervals that contain [mid, mid + 1], and are within the [lo, hi] to improve provided queries.

A query participates in O(log(N)) ImproveViaMid calls.



Starting from subsequence [mid, mid + 1], we expand it to the left, storing all intervals we encounter until we exit the [lo, hi] window.



Left intervals: [12, 15]

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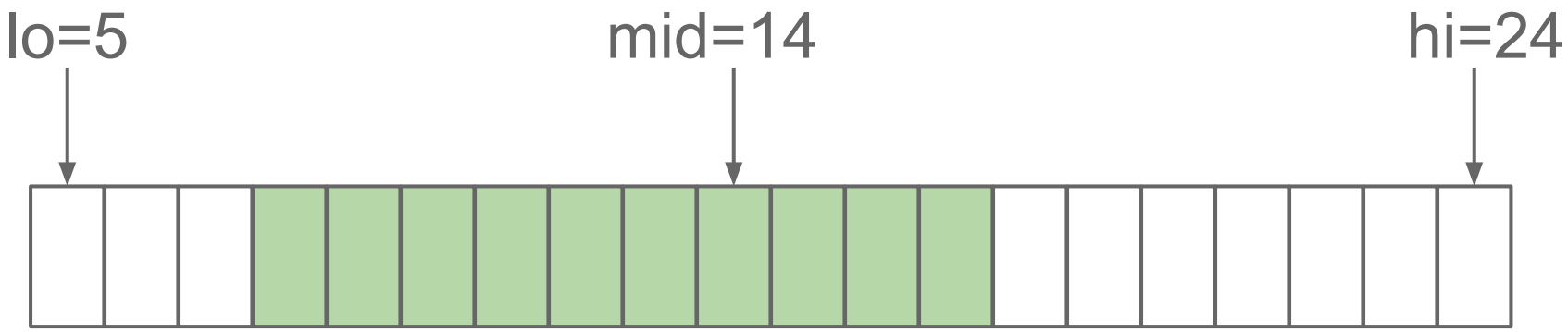

Left intervals: [12, 15], [8, 17]

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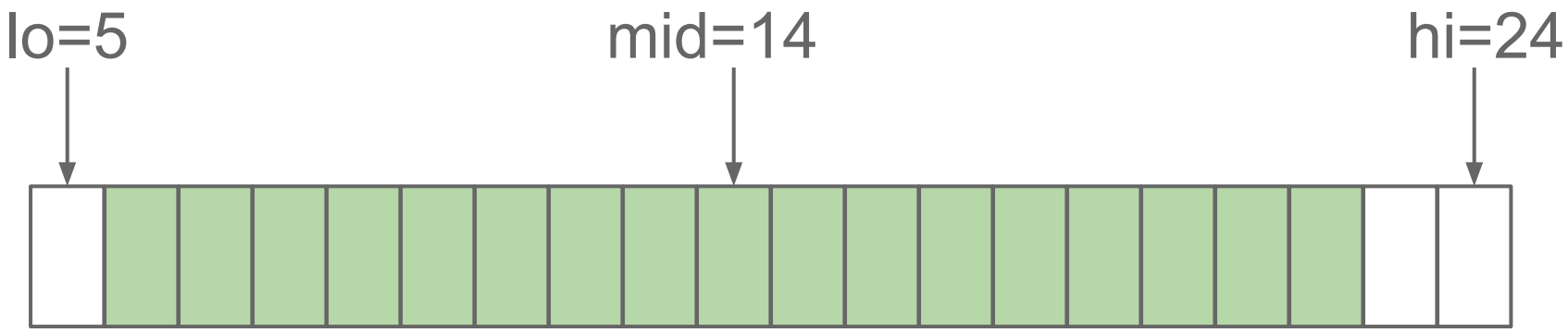

Left intervals: [12, 15], [8, 17], [6, 22]

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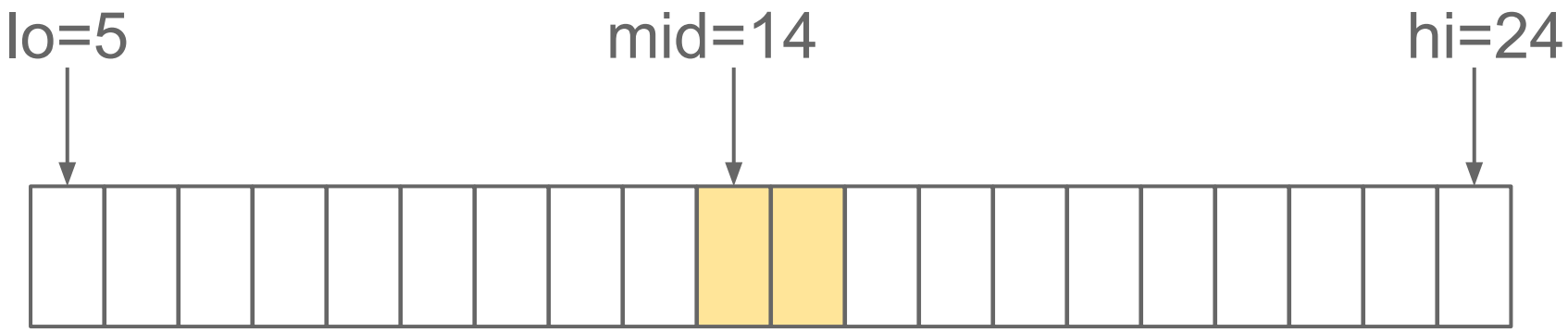

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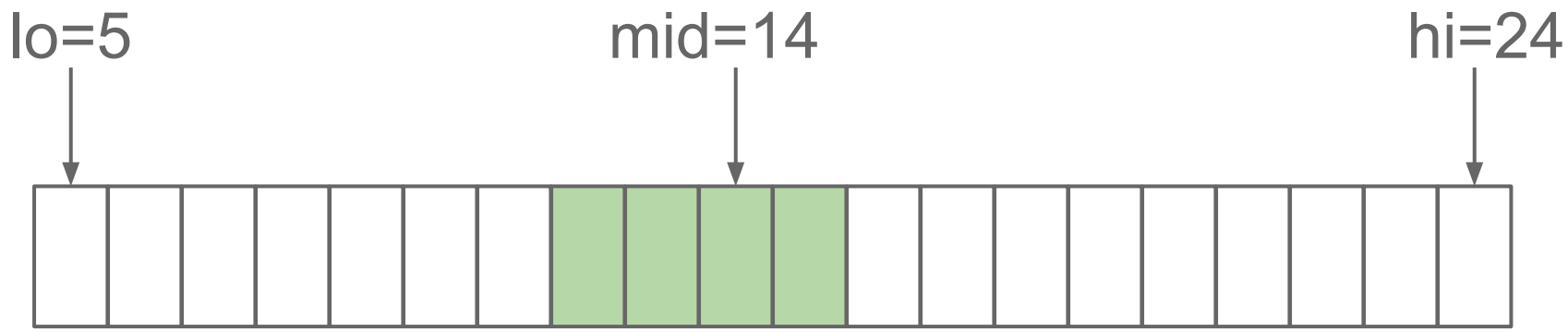
Again, starting from subsequence [mid, mid + 1], we expand it to the right, storing all intervals we encounter until we exit the [lo, hi] window.



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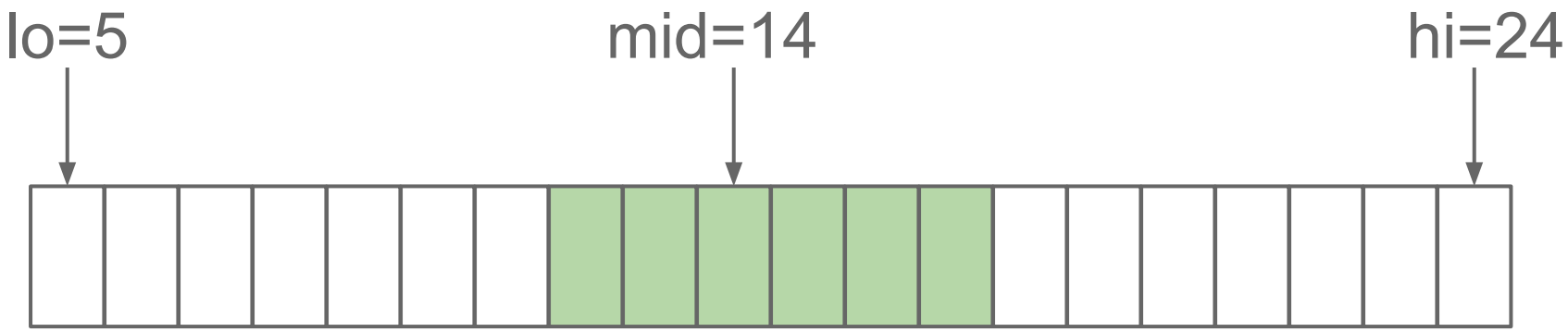

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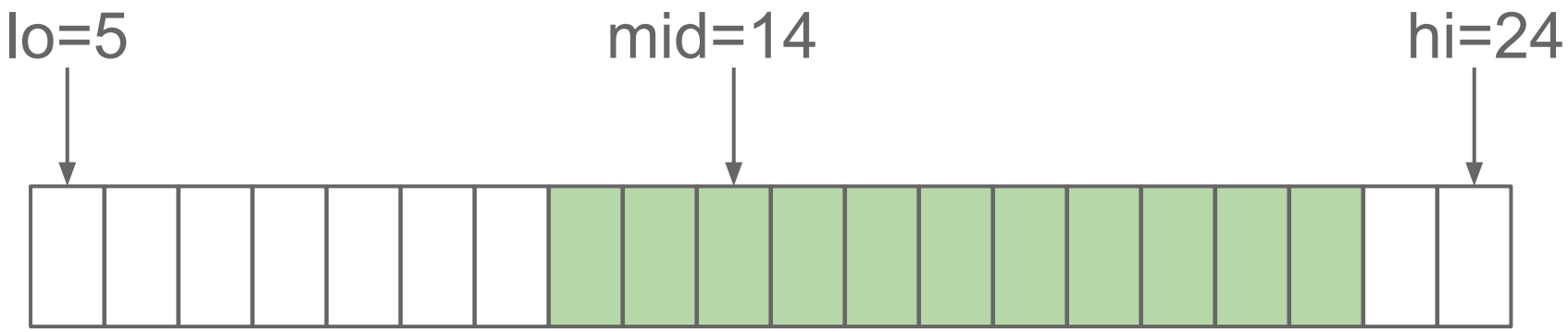

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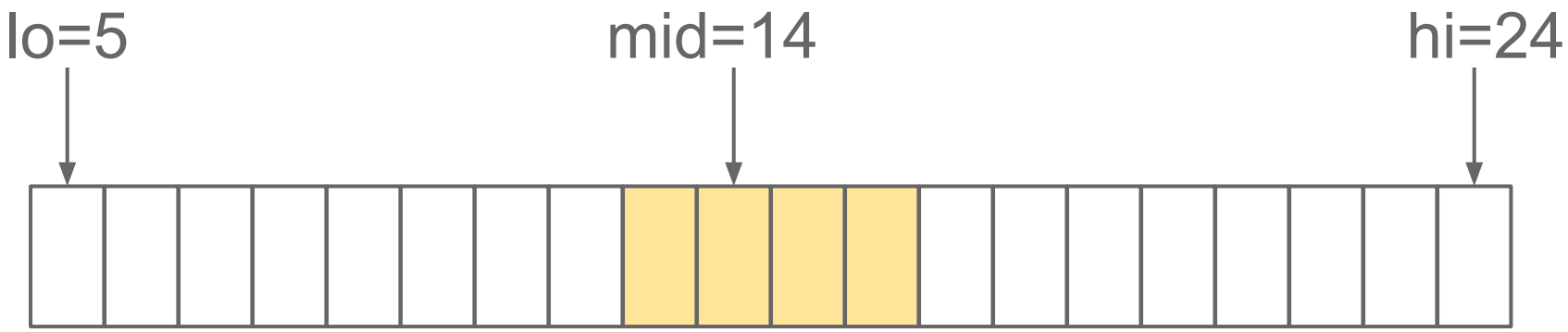

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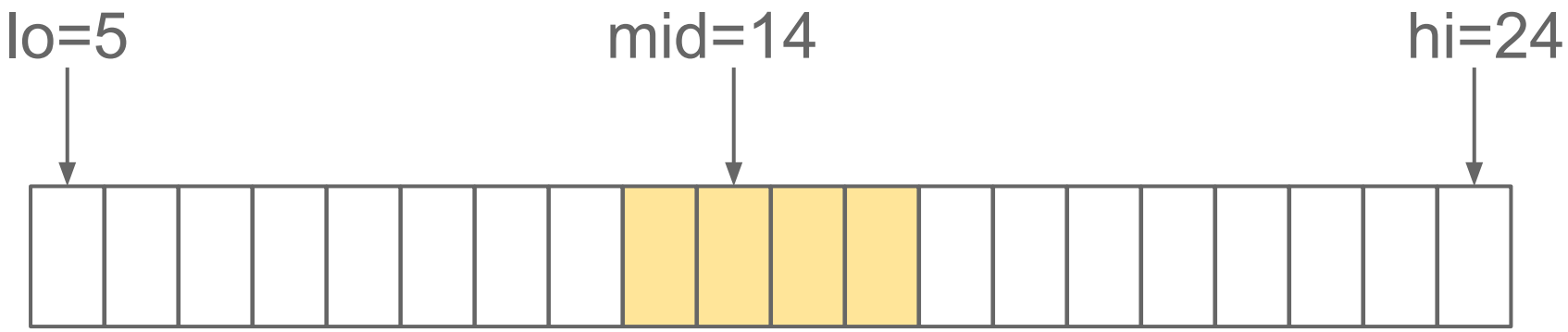
Right intervals: [12, 15], **[12, 17]**, [12, 22]

Finally, for each query [a, b] we find the smallest left interval that contains it and the smallest right interval that contains it. The union of these two intervals is the smallest interval within [lo, hi] that contains the query.



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Right intervals: [12, 15], **[12, 17]**, [12, 22]

We can implement ImproveViaMid(queries, lo, mid, hi) in O(|hi - lo| + queries.size()), for overall complexity of O((N + Q) log N).